M.Sc.-III/Mathematics-305ME/18

M.Sc. 3rd Semester Examination, 2018

MATHEMATICS

(Mathematical Moding of Dynamical Systems)

Paper : 305ME

Course ID : 32155

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

. Answer *any five* questions. 8×5=40

1. (a) For any $n \times n$ matrix A, prove that $\frac{d}{dt}(e^{At}) = A\, e^{At}$.

(b) Suppose $D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$, a diagonal matrix. Then prove that $e^{Dt} = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}$.

(c) For any identity matrix $A = I$, prove that $e^{At} = \begin{bmatrix} e^t & 0 \\ 0 & e^t \end{bmatrix} = I.\, e^t$. 2+3+3=8

2. (a) State Poincare Bendixon Theorem.

(b) Show that $\ddot{x} + p(x)\,\dot{x} + q(x)x = 0$, where $p(x)$ and $q(x)$ are smooth functions and $p(x) > 0, x \in \mathbb{R}$ has no periodic orbit.

(c) Show that, $\dot{x} = y + \frac{x}{4}(1 - 2r^2)$

$$\dot{y} = -x + \frac{1}{2}y(1 - r^2)$$

where $r^2 = x^2 + y^2$ has at least one periodic solution. 1+3+4=8

3. (a) Define Critical point.

(b) Characterize the critical point(s) for the system $\frac{dJ}{dt} = \begin{bmatrix} \lambda & 0 \\ \gamma & \lambda \end{bmatrix} J$, where λ and γ are real.

(c) Draw the phase diagram of the above system. 1+4+3=8

4. A predator – prey population model is given by

$$\frac{dN}{dt} = \gamma N\left(1 - \frac{N}{K}\right) - \frac{\alpha NP}{P + \alpha N},$$

$$\frac{dP}{dt} = \frac{e\alpha NP}{P + \alpha N} - \upsilon P,$$

where N is the prey population and P is the predator population; γ, k, α, e and υ are positive constants.

(a) Explain predator-prey model in terms of ecological systems.

(b) Find the critical points.

(c) Discuss the local asymptotical stability of the system. 2+2+4=8

5. (a) Define saddle node bifurcation.

(b) Consider the one-dimensional system of differential equation $\dot{x} = \mu - x^2$, where μ is a parameter.

(i) Determine the Critical point(s).

(ii) Discuss the bifurcation around the Critical points.

(iii) Draw the bifurcation diagram. 1+(1+3+3)=8

6. (a) Define Hyperbolic Critical point.

(b) For what values of α will the zero solution be stable for the autonomous system of differential equations,

$$\dot{x} = \alpha x + y$$
$$\dot{y} = -x$$

2+6=8

7. (a) Prove that the equilibrium of the following system

$$\dot{x} = y - f(x)$$
$$\dot{y} = -g(x)$$

is stable under appropriate condition by Lyapunov's 2nd method.

(b) Consider the system of differential equations

$$\dot{x_1} = -2x_2 + x_2 x_3,$$
$$\dot{x_2} = x_1 - x_1 x_3,$$
$$\dot{x_3} = x_1 x_2.$$

Discuss the stability of the system by using Lyapunov method. 4+4=8

8. (a) Find the stability at origin for the system $\dot{x} = Ax$, where A is given by $A = \begin{bmatrix} \lambda & -2 \\ 1 & \lambda \end{bmatrix}$, λ is real.

(b) Write down the modified Lotka-Volterra model and find their equilibrium points. Also, discuss the stability of the system. 3+5=8
